1. Consider the system described as:

\[
\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -6 & 5 \\ 5 & -10 & 4 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} u(t)
\]

\[
y(t) = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)
\]

a) Enter the system matrices and create the system description using the `ss` command.

b) Find the transfer function matrix (TFM) of the system (i) using the `ss2tf` command and (ii) using the `tf` command (you should enter the system matrices in case (i) and the system description obtained in part a in case (ii) as arguments). Do you obtain the same TFM in both cases?

c) Find the poles of the transfer function found in part a, (i) using the `roots` command (for the denominator polynomial) and (ii) the `pole` command (for the system description). Also find the eigenvalues of the dynamics matrix of the system using the `eig` command. Compare the three sets of values. Are they the same?

d) Find a realization (the matrices of a state-space description) of the transfer function found in part (i) of part b, using the `tf2ss` command. Compare these matrices to the original system matrices. Are they the same? If not, what kind of a relation exists between the two sets of matrices? Find the similarity transformation matrix \( T \) which converts the given representation to the representation found here. Note that if \( A_2 = TA_1T^{-1}, B_2 = TB_1, C_1 = \begin{bmatrix} B_1 & A_1B_1 & A_1^2B_1 \end{bmatrix} \), and \( C_2 = \begin{bmatrix} B_2 & A_2B_2 & A_1^2B_2 \end{bmatrix} \), then \( C_2 = TC_1 \). Thus, if \( C_1 \) is invertible, then \( T = C_2C_1^{-1} \). The matrix \( C_i \) is called as the controllability matrix for the system with the dynamics matrix \( A_i \) and the input matrix \( B_i \) (\( i = 1, 2 \)).

e) Obtain a plot of the step response of the system, for \( 0 \leq t \leq 5 \), using the `step` command.

f) Let \( x(0) = 0 \) and \( u(t) = 1 \), for \( t \geq 0 \). Obtain a plot of \( y(t) \) for \( 0 \leq t \leq 5 \) using the `lsim` command. Compare this plot to the plot of part e. Are the two plots same? Why?

g) Obtain a 3×1 vector whose elements are random numbers uniformly distributed between -1 and 1 using the `rand` command. Then, repeat part f using this vector as \( x(0) \).

2. Repeat Question 1 for the system described as:

\[
\dot{x}(t) = \begin{bmatrix} -1 & 0 & 5 \\ 0 & -6 & -10 \\ 0 & 5 & 4 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} u(t)
\]

\[
y(t) = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)
\]

In part d, you can use the observability matrix \( O_i = \begin{bmatrix} C_i \\ C_iA_i \\ C_iA_i^2 \end{bmatrix} \), for the system with the dynamics matrix \( A_i \) and the output matrix \( C_i \) (\( i = 1, 2 \)), rather than the controllability matrix (since the controllability matrix is not square in this case), to determine \( T \). In parts f and g, use \( u(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \), for \( t \geq 0 \). Compare your results to the corresponding results of Question 1. Are there any similarities? If so, what are the similarities and why do they happen?