1. Basic matrix operations using the interactive mode of MATLAB: Do the following using MATLAB.

a) Enter the following matrices:

\[ A = \begin{bmatrix} 3 & -3 & 1 \\ 4 & -5 & 2 \\ 4 & -5 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 4 & -2 \\ -2 & 5 & -2 \\ -1 & 2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \]

b) Find the eigenvalues and eigenvectors of \( A \) and of \( B \) (use the \texttt{eig} command). Also find the rank of \( A \) and of \( B \).

c) Find the product of \( A \) and \( B \) and of \( B \) and \( A \). Also find the product \( AC \) and \( BAC \). Can you find the product \( CB \) (try it)? Why?

d) Find \( A' + B \) (\( \cdot' \) indicates the transpose of \( \cdot \)).

e) Find \( B^{-1} \) (use the \texttt{inv} command) and find \( X = B^{-1}C \). Verify that \( X \) satisfies \( BX = C \). Can you find \( A^{-1} \) (try it)! Why? (Recall the result of part b).

f) Type \( A(1:2, 2:3) \) and \( B(1,:) \). What do you obtain in each case? Obtain a vector whose elements are the second column of \( A \).

2. Programming mode of MATLAB: Write a MATLAB program to calculate \( x(n) = \sum_{k=0}^{n} C' B^k C \) for \( n = 0, 1, \ldots, N \) (or for \( n = N, N + 1, \ldots, 0 \) if \( N \) is negative, in which case the sum must be taken from \( k = n \) to \( k = 0 \)) and plot \( x(n) \) vs. \( n \). Here, \( B \) and \( C \) are as in Question 1 and \( N \) is read from the input terminal using the \texttt{input} command at a prompt “Enter \( N > \)”. When \( k \) is negative, \( B^k \) means \( (B^{-1})^{(-k)} \); and \( B^0 \) is the identity matrix, by definition. Your program must produce an error message if \( N \) is not an integer. Run this program for \( N = 10, N = -10 \), and for \( N = 0.5 \).