EEM 342: Fundamentals of Control Systems
2006-2007 Spring Semester Final Exam
Time: 120 minutes; Total Points: 100

One size-A4 formula sheet and a hand calculator, not capable of symbolic operations, may be used.

Solve either
• Questions 0, 1, 2, and 3 only, or
• Questions 0, 1, and 4 only.

0. (2 points) Write the group you are registered to on the top-right corner of your answer sheet.

1. Consider the system shown below. The mechanical system shown on the left consists of a body of mass $M$, which is connected to a fixed point by a linear spring of spring constant $k$ and a damper which provides viscous friction of coefficient $\beta$. The body can move back and forth in one direction on a frictionless surface. The displacement of the body from its equilibrium is denoted by $q$. The voltage applied to the electrical circuit shown on the right is proportional to the velocity, $\dot{q}$, of the body. The proportionality constant is $\alpha$. The force applied to the body, $u$, forms the input of the system. The output of the system is $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, where $y_1$ is the current through the circuit and $y_2$ is the voltage across the inductor.

![Mechanical System Diagram](image1)

![Electrical Circuit Diagram](image2)

a) (15 points) Find the transfer function matrix from $u$ to $y$.

b) (15 points) Choose a state vector for this system and write the state equations.

2. (28 points) Consider the sampled-data system whose block diagram is given below, where the two samplers of period $T$ are synchronous. ZOH indicates a zero order hold and UD indicates a unit delay (a delay element of delay time $T$). Find all values of $K$ (real) and $T$ (real and positive) for which this system is stable. Indicate those values on the $K$ vs. $T$ plane.

![Sampled-Data System Diagram](image3)
3. Consider the linear time-invariant (LTI) continuous-time plant with transfer function
   \[ G(s) = \frac{2}{s^2 + 3s + 2} \]. It is desired to design a controller for this system such that the plant
   output can track constant reference signals with no steady-state error and no overshoot happens in the transient response.

   a) (5 points) Is it possible to satisfy the given requirements using a proportional (P) controller? Why?
   b) (15 points) Design a continuous-time proportional-integral (PI) controller to satisfy the given requirements and to obtain as fast response as possible (i.e., settling time must be as small as possible). Draw a block diagram showing the implementation of the controller.
   c) (5 points) For the closed-loop system obtained in part b, calculate the steady-state error in response to a unit ramp reference.
   d) (15 points) Design a continuous-time proportional-integral-derivative (PID) controller to reduce the steady-state error in response to a unit ramp reference to 10% of the value found in part c, without violating the stated requirements and without slowing the response obtained in part b. Draw a block diagram showing the implementation of the controller.

4. (68 points) Consider linear time-invariant (LTI) continuous-time plants with transfer function
   a) \( G(s) = \frac{2s}{s^2 + 3s + 2} \) b) \( G(s) = \frac{s^2 + 1}{s^2 + 3s + 2} \)
   It is desired to design a LTI controller, to be implemented on a digital computer, such that the closed-loop system is internally stable and the output of the plant can track ramp reference signals with no steady-state error. For each plant, determine whether it is possible to design such a controller. If it is not possible, explain the reason. If it is possible, then
   
   either:
   
   First design a continuous-time controller to achieve the requirements. Draw a block diagram for the continuous-time implementation. Then, choose an appropriate sampling period (explain how you choose it) and find the equivalent discrete-time controller. Also write a computer program to implement this controller on a digital computer and draw a block diagram showing the implementation of the actual system.
   
   or:
   
   Choose an appropriate sampling period (explain how you choose it) and find the discrete-time equivalent of the plant (which must, of course, be preceded by a D/A converter which includes a ZOH). Then, design a discrete-time controller to achieve the requirements. Also write a computer program to implement this controller on a digital computer and draw a block diagram showing the implementation of the actual system.